On the cooperation of the 20th century mathematations and another topics.

TAKESHI KANO

Prologue

One of the reasons to write this paper, is an NHK's TV talk by Prof. S. YAMANAKA, who received the Nobel Physiology prize because he found the I-S cell, where he said " Japanese scientists must be more international and shall extend to coopeate with many foreign scientists. But unfortunately, almost Japanese scientists have no sufficient ability to speak and hear foreign language, especially English. This fact is the most weak point. " I also realize this fact because since 1971 I visited many foreign countries e.g. Soviet Union (now Russia), China England, German, France and USA, etc.. Through such experiences I learnd the true speaking - English was quite different from that of Japanese English. For an example, I first learned

the x^n, for an example. It reads as " x to the n".
These facts necessarily show the exclusive education of Japanese elementary level. I would like to indicate later such facts that come from ignorance and misunderstanding in most cases.

Chapter I

In this chapter, I introduce first the famous Book " The Scottish Book " (Mathematematics from the Scottish Café) Edited by R. D. Mauldin, Birkhäuser, Boston. Basel. Stuttgart. 1981 in Birkhäuser Boston. The picture of the Café is still now situated near the University Lwōw, Poland.

In "Anedotal History of the Scottish Book", S. Ulam says as following: Most of the problems are due to a few local mathematicians, myself included. Actually, many of the ealier problems originated well before 1935 — perhaps 6 or 7 years before — during the period when I was still a student. ..... I was then able to take part in the informal discussions — generally among two or three of us at a time — which were a standard feature of mathematical life in pre-World War II Lwów. .... ultimately M. Kac made his appearance, and I lost my position to him, my jnior by some five years. The story of the Scottish Book could also be called the "Tale of Two Coffee Houses", the Café Roma and, right next to it, the Café Szkocka or Scottish Café. .... The meetings were usully held on Saturday in a seminar room at the University — hence close to the Cafés. The time could be either afternoon or evening. .... but the really fruitful discussinons took place at the Café Roma after the meeting was officically over.

such as Mazur, Orlicz et. cet..

8. (Mazur, prize: five small beers)

(a) Is every series summable by the first representable as a Cauchy product of two converging series? Or else, equivallentally,

(b) Can one find for each convergent sequences $\{z_n\}$ two convergent sequences $\{x_n\}$, $\{y_n\}$ such that

$$z_n = \frac{x_1 y_n + x_2 y_{n-1} + \cdots + x_n y_1}{n} \quad ?$$

13. (Ulam) Let be the class of all subsets of the set of integers. Two subsets $K_1, K_2 \in E$ are called equivalent or $K_1 \equiv K_2$ if $K_1 - K_2$ and $K_2 - K_1$ are at most finite sets. There is given a function $F(K)$ defined for all $K \in E$; its range is contained in E and

$$F(K_1 + K_2) = F(K_1) + F(K_2)$$
$$F(\text{comp. } K) \equiv \text{comp. } F(K).$$

Question: Does there exist a function $f(x)$ (x and $f(x)$ natural integers) such that

$$f(k) \equiv F(k) \;?$$

14. (Schauder, Mazur)

Let $f(x_1, \ldots, f_n)$ be a function defined in the cube K_n.

Let us suppose that f possees almost every-where all the partial derivatives up to the rth order and the derivatives up to the order $(r-1)$ are absolutely continnous on almost every straight line parallel to any axis.

All the partial derivatives (up to the order r) $\in L^p$, $p>1$.

Does there exist a sequence of polynomials $\{w_i\}$ which converge in the mean in the pth power to f and in all partial derivatives up to the order r? For $r=1$ this was settled positively by the authors. An analogous problem exists for domains other than K_n.

28. (Mazur; prize: Bottle of wine)

Let

(*) $\sum_{n=1}^{\infty} a_n$

be a series of terms and let us denote by R the set of all numbers a for which there exists to a. Is it true that if the set R contains more than one number but not all nubers, then it

165. (Ulam; prize: two bottles of wine)

Let be a sequence of rational points in the n-dimentional unit sphare. The first N points $p_1, \ldots, p_N$ are transformed on N points (also located in the same sphere) $q_1, \ldots, q_N$ all different. We define a transformation on the points p_n, $n > N$, by induction as follows: Assume that the transformation is defined for all points p_ν $(\nu < n)$, and their images are all different. This mapping has a certain Lipschitz constant L_{n-1}. We denote the inverse mapping by L'_{n-1}. We define the mapping at the point p_n so that the sum of the constants $L_n + L'_n$ should be minimum. (In the case where we have several points satisfying this postulate we select one of them arbiturarily)

Question: Is the sequence $\{L_n + L'_n\}$ bounded?

120 (Orlicz)

Let x^{n_i} be a sequence of powers with integer exponents on the in the interval (a, b) and

$$\sum_{i=1}^{\infty} \frac{1}{n_i} = +\infty$$

Give the order of approximation of a function satisfying a Hölder condition of polinomials;

$$\sum_{i=1}^{p} a_i x^{n_i}.$$

121 (Orlicz)

Given an example of a trigonometric series

$$\sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

everywhere divergent and such that

$$\sum_{n=1}^{\infty} (a_n^{2+\varepsilon} + b_n^{2+\varepsilon}) < +\infty$$

for any $\varepsilon > 0$.

151 (WAVRE: Prize: "fondue" in Genova")

Does there exist a harmonic function in a region which contains a cube in its interior, which vanishes on all the edges of the cube?

One does not consider $f \equiv 0$.

COMMENT BY A. ZYGMUND

The origin and history of the Scottish Book is described by Prof. Ulam in his own lecture and I could not add much here.

The book is a product of one of the mathematcal schools in Poland, that of Lwów, while I myself, born and educated in Warsaw, belonged to what was then known, both in Poland and abroad, as the Warsaw mathematical school.

There was a close collaboration between individuals of both schools, and though my personal contact with Lwów was rather loose, I was very much interested in the work going on my work.

. . . . . .

The school of Lwów is technically no longer in existence and its organ *Studia Mathematica*, began in 1932, is now being published 波文字 in Warsaw. But the influence of the work of its founders and their pupils continues and grows in various Polish mathematical centers. The names, of Banach, Steinhaus, Schauder, Kaczmarz, Auerbach, Ulam, Mazur, Orlicz, Nikliborc, Schreier, Ruziewicz, Kac, and others symbolize achievements of this school.

mathematical problem book entitled as

"Hungarian Problem Book"

which is based on the Eötvös Competitions,
1894 – 1905 (revised and edited by G. Hajós,
G. Neukomm, J. Surányi, originaly completed
by J. Kürschák ; published by Random House
and The L. W. Singer Company ; New York, Syracuse.
copyright, 1963, by Yale University)

This book has a similar common property as
"The Scottish Book". Now I would like
to show the only small part of "Editors' Note",
then after that certain problems extended in
this book.

" The publication of distinguished problem collections
in the New Mathematical Library has been one of
the aims of the Monograph Project of the School
Mathematics Study Group from the begining.

– – –

We are grateful to Professors Hajós, Neukomm +
and Surányi.

(+ Prof. Neukomm died in 1957)

To conform to the design of the New Mathematical Library, we had to publish the contest problems from 1894 – 1928 in two volumes. If these turn out to be as useful as we hope, we shall probably publish the problems from 1929 to date as well. (author's comment: as far as I know, this was not realised)

The reader should be aware that this collection is far from routine. While the solutions require elementary mathematical techniques only, a great deal of ingenuity is often necessary. The main purpose of this collection is to instruct the reader by having him (or her; this word added by me) study the solutions prsented here together with some of the more sophisticated material in the explanatory notes. The reder should not feel that he (and she; this word also added by me) is being tested, but that he (and she; same reason) is being taught.

New York 1962

~~Values of x and y~~

<3> The lengths of the sides of a triangle from an arithmetic progression with diference d. The area of the triangle is t. Find the sides and the sides and angles of this triangle. Solve this problem for the case $d=1$ and $t=6$.

1895 Competition

<1> Prove that there are $2(2^{n-1}-1)$ ways of dealing n cards to two persons (The players may receive unequal numbers of cards)

1899 Competition

<1> Let x_1 and x_2 be the roots of the equation

$$x^2-(a+d)x+(ad-bc)=0.$$

Show that x_1^3 and x_2^3 are the roots of

$$y^2 - (a^3 + d^3 + 3abc + 3bcd)y + (ad - bc)^3 = 0$$

<2> Prove that, for any natural number n, the expression

$$A = 2903^n - 803^n - 464^n + 261^n$$

is divisible by 1897.

1901 Competion

<1> Prove that, for any positive integer n,

$$1^n + 2^n + 3^n + 4^n$$

is divisible by 5 if and only if n is not divisible by 4.

<2> Let a and b be two natural numbers whose greatest common divisor (g.c.d) is d. Prove that exactly d of the numbers

$$a,\ 2a,\ 3a,\ \dots,\ (b-1)a,\ ba$$

are divible by b.

$x = \sin\alpha$, $y = \sin\beta$, there can be four different values of $z = \sin(\alpha+\beta)$.

(a) Set up a relation between x, y and z not involving trigonometric functions or radicals.

(b) Find those pairs of values (x, y) for which $z = \sin(\alpha+\beta)$ takes on fewer than four distinct values.

1905 Competition

<1> For given positive integers n and p, find necessary and sufficient conditions for the system of equations

$$x + py = n, \quad x + y = p^2$$

to have a solution (x, y, z) of positive integers. Prove also that there is at most one such solution.

Chapter III

As I have noted in the Prologue in this last chapter I want to show the difference between real English and Japanese-using-English. Also would like the difference betwaen U.K. and U.S..

(1) <Japan> AM 10, PM10

<Emlish> 10 a.m. 10 p.m.

(2) <Japan> morning service (means breakfast at hotel)

<English> worship at a shrine

(3) <Japan> reform (means to change old parts, house interior et. cet.)

<English> renovation

(4) <Japan> handle

<English> steering-wheel

(5) <Japan> cooler

<English> air-conditioner

(6) <Japan> repeater (means to visit fixed shop)

<English> criminal who does plurally

<U.K.> lift <U.S. and Japan> elevator

<U.K.> mobile phone <U.S.> cell phone

<U.K & E.U.> foot-ball

<U.S. & Japan> succer